EE120 Lecture 1: Introduction

Time $\longleftrightarrow$ Frequency

- continuous time

(CT)

$x : \mathbb{R} \longrightarrow \mathbb{R}$ signals $\in$ fcns

domain

fcn

codomain

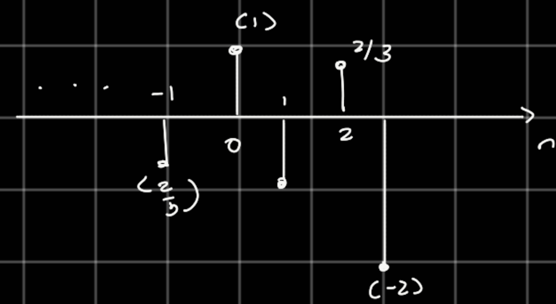
rule that maps domain to codomain (tells you what to do w/ each element in the domain)

- discrete time signal

$x : \mathbb{Z} \longrightarrow \mathbb{R}$ real-valued DT signal

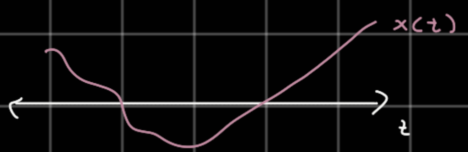
or

$\mathbb{C}$ complex-valued DT signal

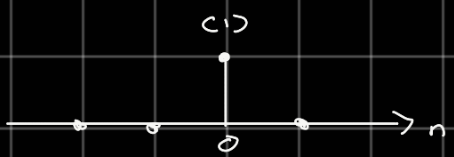


$x(0) = 1$

$x(n) = \frac{2}{3}\delta(n+1) + \delta(n) - \delta(n-1) + \frac{2}{3}\delta(n-2) - 2\delta(n-3)$



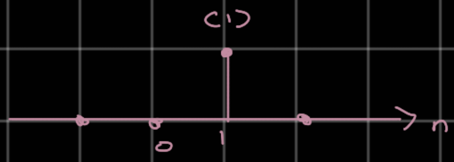
CT signal



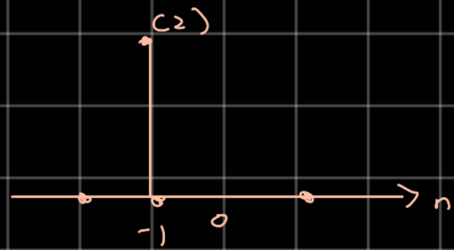
$$\delta(n) = \begin{cases} 1 & n = 0 \\ 0 & o/w \end{cases}$$

DT unit impulse

(Kronecker Delta)



$$\delta(n-1) = \begin{cases} 1 & n=0 \\ 0 & o/w \end{cases}$$



$$2\,\delta(n+1)$$

→ can express any DT signal in terms of the Kronecker delta

$$\boxed{x(n) = \sum_{k=-\infty}^{\infty} \overbrace{x(k)}^{\text{value at}}\, \underbrace{\delta(n-k)}_{\text{k-shifted impulse}}}$$

DT Decomposition of Signals

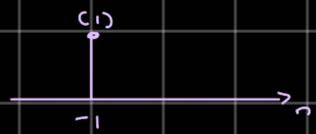
$\delta(2n+2)$ — note that this yields the same plot as $\delta(n+1)$ but if you replace one of the pink numbers with another scalar it would be undefined

↳ why?

→ eg. set one of the values to 3: $\delta(3n-2)$

Where should this be 1? At $3n-2=0 \Rightarrow n = -\frac{2}{3}$

Problem: we are dealing with discrete values & can't have fractions ⇒ the plot would be 0 ∀ values



Systems

$$\underbrace{x}_{\text{input signal}} \rightarrow \underbrace{\boxed{H}}_{\text{system}} \rightarrow \underbrace{y}_{\text{output signal}}$$

$y = H(x)$

↳ a system is a function where the domain is a signal

↳ X: input signal space (domain of H) $x \in X$

↳ Y: output signal space $y \in Y$

$$H: X \to Y$$

- for a DT system, $\underbrace{X = [\mathbb{Z} \to \mathbb{R}]}_{\text{set (space) of real DT signals}}$ & $Y = [\mathbb{Z} \to \mathbb{R}]$

- for a CT system, $X = [\mathbb{R} \to \mathbb{R}]$ & $Y = [\mathbb{R} \to \mathbb{R}]$

Time Invariance :

if

$$\underbrace{x(n)}_{\text{value of signal at time/sample } n} \longrightarrow \boxed{H} \longrightarrow y(n)$$

- we say H is time-invariant if

$$\hat{x}(n) = x(n - \underbrace{N}_{\substack{\text{shift} \\ N \in \mathbb{Z}}}) \longrightarrow \boxed{H} \longrightarrow \hat{y}(n) = y(n-N)$$

and this holds $\forall x \in \mathcal{X}$ & $\forall N \in \mathbb{Z}$

Linearity

Given

$$x_1 \longrightarrow \boxed{H} \longrightarrow y_1$$

$$x_2 \longrightarrow \boxed{H} \longrightarrow y_2$$

, linearity gives you the superposition principle:

$$\alpha_1 x_1 + \alpha_2 x_2 \longrightarrow \boxed{H} \longrightarrow \alpha_1 y_1 + \alpha_2 y_2$$

$$\alpha_1, \alpha_2 \in \mathbb{R}$$

Linear - Time Invariant Systems (LTI)

- if a system has linearity and time-invariance properties, we call it a linear time-invariant (LTI) system

$$\underbrace{\delta(n)}_{\text{impulse}} \longrightarrow \underset{\text{LTI}}{\boxed{H}} \longrightarrow \underbrace{y(n) = h(n)}_{\text{Impulse Response of } H}$$

the beauty of LTI systems

- Claim: if I know impulse response of a system, I can determine H's response to an arbitrary input

↳ Why?

Recall: $x(n) = \sum_{k=-\infty}^{\infty} x(k)\,\delta(n-k)$

$$\delta(n) \longrightarrow \boxed{H} \longrightarrow h(n)$$

$$\delta(n-k) \longrightarrow \boxed{H} \longrightarrow h(n-k) \qquad \text{TI}$$

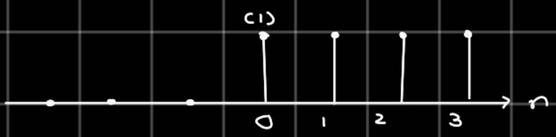
$$x(k)\,\delta(n-k) \longrightarrow \boxed{H} \longrightarrow x(k)\,h(n-k) \qquad \text{Scaling Property of Linearity}$$

$$x(n) = \sum_{k=-\infty}^{\infty} x(k)\,\delta(n-k) \longrightarrow \boxed{H} \longrightarrow y(n) = \sum_{k=-\infty}^{\infty} x(k)\,h(n-k) \qquad \text{Additivity Property of Linearity}$$

Example of DT LTI system

$$x(n) \rightarrow \boxed{H} \rightarrow y(n) = \frac{x(n)+x(n-1)}{2}$$ } Two-point moving average

Discrete Time Unit Step



$$u(n) = \begin{cases} 1 & n \geq 0 \\ 0 & n < 0 \end{cases}$$

$$u(n) = \delta(n) + \delta(n-1) + \delta(n-2) + \cdots$$

$$u(n) = \sum_{k=0}^{\infty} \delta(n-k)$$

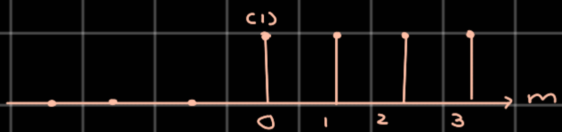
- represents suddenly applied excitation/input to a system

change of variables

$$m = n - k$$

$$u(n) = \sum_{k=0}^{\infty} \delta(m)$$

different way of viewing this eqn:
- DT version of integration



DT - LTI

$$x(n) \rightarrow \boxed{\begin{matrix} H \\ u(n) \end{matrix}} \rightarrow y(n) = ?$$

$$\delta(n) \rightarrow \boxed{H} \rightarrow h(n) = u(n)$$

$$y(n) = \sum_{k=-\infty}^{\infty} x(n) \underbrace{h(n-k)}_{u(n-k) = \begin{cases} 1 & n-k \geq 0 \Rightarrow k \leq n \\ 0 & k > n \end{cases}}$$

$$y(n) = \underbrace{\sum_{k=-\infty}^{n} x(k)}_{\text{DT counterpart to integration}}$$